1. **Choose your method.** Let R be the region bounded by the given curves. Use the method of your choice to find the volume of the solid generated by revolving R about the given axis.

   (a) $y = x - x^4$, $y = 0$; about the x-axis.

   (b) $y = x - x^4$, $y = 0$; about the y-axis.

   (c) $y = \sin x$, $y = 1 - \sin x$, for $\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$; about the x-axis.

   (d) $y = \sqrt{\ln x}$, $y = \sqrt{\ln x^2}$, $y = 1$; about the x-axis.

2. **Finish the examples from Section 6.5.** Find the length of the given curves

   (a) $y = \frac{1}{2}(e^x + e^{-x})$ on $[-\ln 2, \ln 2]$

   (b) $y = f(x) = \ln(x + \sqrt{x^2 - 1})$ on the interval $[0, \sqrt{2}]$.

3. **A logarithmic property.** Show that the length of the curve $y = \ln(f(x))$ for $a \leq x \leq b$ is

$$L = \int_a^b \frac{\sqrt{(f(x)^2 + (f'(x))^2)}}{f(x)} dx \tag{1}$$

Use Equation (1) to find the length of the curve $y = \ln(\cos x)$ on $[0, \frac{\pi}{4}]$.

4. **Area = arc length.** The hyperbolic sine and hyperbolic cosine are defined over all real numbers as

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$

One defining characteristic of the hyperbolic functions is their relation to the hyperbola $x^2 - y^2 = 1$, particularly that $\cosh^2 x - \sinh^2 x = 1$.

Show that the area under the curve $y = \cosh x$ from $x = 0$ to $x = a$ is equal to the length of the same curve on the same interval $[0, a]$.

HARD: Can you find all non-negative, continuous, differentiable functions $f(x)$ such that the area under graph of f and the arc length of $y = f(x)$ coincide on $[0, a]$ for all $a > 0$?